Calculation of PLT near field from domestic wiring

RM Harris

1. Contents

2. Revision History

Issue	Date	Page	Remarks
0.1	14th Jan 2017	1 - 8	First Draft
0.2	16th Jan 2017	1 - 40	Tidying up
0.3	27th Feb 2017		Typos

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3. Purpose

This document records a first attempt at a theoretical model of a domestic PLT wiring harness, from which EMC criteria may be derived. In particular, the near field from a finite straight mains power line is calculated for a low-frequency (LF) common-mode, uniform current excitation

4. Disclaimer

No liability shall be accepted for any statement, implied or explicit, or any inference, or promise of results.

5. Definitions

A full list of abbreviations can be found in Appendix A

Much of the analysis involves vector calculus, and so to clarify the mathematical passages, the following advance explication is offered. Spatial vectors are represented by bold capitals, e.g. **A**. When the emphasis is on the magnitude of the vector quantity, the un-emboldened capital is used, e.g. A. When the attention is on its behaviour with respect to a particular spatial variable (the x-axis) or other parameter, it may be represented as A(r), or A = A(x), etc.

The magnitude of a vector is indicated by the modulus signs, e.g. | A |.

There is no font sign for the “hat” operator, and so the following invention will stand in for the unit vector indicating “direction of”, thus: dir [**A**] = **A**/(| A |).

The cross product is represented by the caret, **^**, e.g. **R** = **S ^ T** conventionally.

The usual meanings are attached to the straight and curly “delta” signs, e.g. dZ(x)/dx, and ∂Z(x,y)/∂x, standing for direct, and partial derivatives respectively.

Some *ad hoc* expressions may be used to avoid wordy explanations, e.g. “x = x(P)” means “the x-(abscissa) co-ordinate of the point P in space.

In E/M theory, the convention is to use emboldened square **[ ]** braces to imply retarded time.

6. Related Documents

[1] (no Doc. Code) Electro-magnetic theory applied to EMC issues in PLT < PLT_safe_v8.doc > revision number 0.08 dated 9th October 2004

[2] IEE Ninth International Conference on HF Radio Systems and Techniques, University of Bath 2003

[3] BBC White Paper WHP 064 Digital Radio Mondiale – revitalising the bands below 30 MHz,

[4] BBC White Paper WHP 067 The effects of power-line telecommunications on broadcast reception: brief trial in Crieff *Jonathan Stott and John Saltel*

[5] ITU-R BS703, Annex 1

[6] Electricity and Magnetism BI Bleaney and B Bleaney, Oxford University Press

[7] Electromagnetic Waves and Radiating systems, Edward C Jordan and Keith G Balmain. Prentice-Hall, 2nd edition 1968 (pp 333-336 ff)

[8] The IEE Wiring Regulations, BS 7671:2001 (16th Edition) London.

7. Introduction

Power Line Communications (PLC) or Power Line Transmission (PLT) has been in an immature phase for a decade or two. There is an ETSI Project, *EP PLT*, studying and promoting it, and occasional announcements in the trade press. One of the most startling announcements was in the keynote speech at the IEE's prestigious Ninth International Conference on HF Radio Systems and Techniques in Bath [1] [2], 2003. Jonathan Stott, BBC Research Laboratories, pointed out [3] the impending EMC catastrophe, as two major sets of regulation seem set to collide.

More careful measurement and more realistic theoretical models are called for before the debate gets entrenched. The BBC has made measurements [4] inside houses where Scottish Power were running trials in Crief. The need to calibrate both search coils and corroborate the difficult measurements with modelling has since become apparent.

As with many complex and ill-defined systems and situations, the starting point has to be a simplified element whose properties can be analysed beyond all doubt. Then, and only then, the complex scenario can be built up in stages. In the case of indoor PLC or PLT, the basic element is the magnetic near field that is exerted around a linear run of low-voltage power cable, assumed to be a twisted pair. The dimensions are critical. When the length of run is very large compared to the distance from the point of measurement, then the well-known college physics equations can be used, based on a trivial application of Ampère's Law. When, on the other hand, the magnetic field is to be measured at a great distance, the inverse distance-squared law of Biot and Savart can be applied with reasonable accuracy. For intermediate distances the college formulas are not valid. Another key dimension is the radio wavelength corresponding to the frequency of the common-mode current excited in the power line. At low frequencies, the wavelength is many times the geometry of the house and so the static formulas can be valid. At PLT data rates, the wavelengths are comparable to house dimensions and so the static approximation breaks down.

The BBC approach is to model resonant stubs as monopoles, whereas the approach proposed in this paper is to model a ring main as a small magnetic loop antenna.

The eventual aim is to relate the just-permissible strength of the interfering magnetic field to a frequency mask of common-mode current in a length of power line.

8. Physics of Near Field from a Rectangular Loop

8.1. Relation of loop to domestic wiring

The typical house-wiring layout is a contingent mixture of horizontal rectangular loops with linear spurs running in predominantly vertical directions. The conductor pairs are usually run close together if not actually twisted around each other, so that the only contribution to external E/M fields is due to the magnetic effect of common-mode current. As far as the electrostatic field is concerned, the lighting power circuit usually has one conductor (the neural) close to "earth" potential, whilst for the power circuits there is (in the UK) a mandatory "Earth" conductor in the cable bundle. A domestic PLT driver is expected to use the differential mode, whereby the PLT signal consists in the superimposition of two equal and opposite time-varying electrical potentials applied to the two power conductors. The time-varying electric fields are entirely confined between the two proximate conductors, which act as a species of transmission line with characteristic impedance around 100 - 200 ohms.

The residual external magnetic field is due to the current imbalance between the two conductors, i.e. the common-mode current. The ratio of that imbalance to the proper transmission-line current for a PLT signal is called the Common-Mode Rejection Ratio, CMMR. Whilst ideally the CMMR ought to be zero, in practice it is non-zero because of unequal a.c. impedances to ground. In a ring main, some current may even flow the "wrong" way round a floor circuit. Consumer equipment that is plugged into power sockets may have a.c. decoupling of one side (e.g. the neutral line) to Earth, hugely unbalancing the power pair. Thyristor-mediated speed controls on dimmer switches and power tools may further unbalance the PLT transmission line.

Hence, the general case is best represented by a rectilinear grid of conducting wires carrying variable residual currents related to the normal PLT current *via* the CMMR.

A quasi-static approach will be used, by which is meant a formulation that is equally applicable to constant (d.c.) and to low-frequency time-varying currents (a.c.). The formulation is later transformed for r.f. currents, the distinction between a.c. and r.f. here being that in the former case the free-space wavelength is very much larger than the scale of the house, whereas in the latter case, the wavelength is comparable with actual wiring runs.

Without prejudice to the r.f. case, the treatment of d.c. (i.e. in magnetostatics) requires closed circuits. The well-known Law of Biot and Savart is based on calculus and produces results that are contrary to commonsense if the integration is not completed around a closed curve. (Solutions to problems involving open-ended conductors always require r.f. analysis; the stub is approximated to an antenna of some sort.) Ergo, the atomic element for this PLT analysis is the (closed) rectangular loop, and not a single evenly conducting element.

The loop will be termed a "magnetic loop" to signify that electrostatic fields will be treated as non-existent, as though the loop had zero impedance. That such is believed to be realistic has been borne out by other researchers in the field [8].

The analysis of the magnetic loop may be taken from d.c. directly to low-frequency a.c., and then transformed using the classical method of (Liénard-Wiechert) retarded potentials into the r.f. case. The analysis assumes nothing special about the spatial scale, giving a general result for field locations close to the wires and for much farther away too. On the other hand, the purely r.f. analysis of the open-ended conductor (stub) cannot be taken down in frequency to give a corresponding a.c. or d.c. analysis.

Once the general result for a rectangular loop has been established, the principle of superposition may be invoked to calculate d.c., a.c. or even r.f. near fields inside a complex of such loops, taking due care with the orientations of loops and field vectors.

8.2. Optimising the loop geometry

From the point of view of suffering radio interference, the worst places to be are inside a particular current loop. There are textbook formulas for the magnetic field at the centre of a circular coil and the rectangular loop is not so different. The following analysis tackles the magnetic field outside a rectangular loop, at all points along a perpendicular bisector on one side. In so doing it takes a slightly more benign view of PLT interference. It is also arguably the more general case since, in a large mesh, the point of interest, P, is in the centre of only one loop whereas it is on the outside of many.

The loop is considered to comprise the four linear elements as depicted in **Figure 1**, namely AB, BC, CD and DA. The point of interest, P, is on the perpendicular bisector of the long side, AD. Side AC represents the proximate power line carrying current **I** (or time-varying current, **I**(t)). The same current flows round the loop along short side AB, rear long side BC, and short side CD. The magnetic field, **H** = **H**(x) (into the paper) is dominated by the current contribution from conductor DA, the two short sides contributing smaller components that, being out of the paper, tend to cancel the effect of DA. The rear side, BC, more evidently acts in opposition to the effect of DA. Due to the symmetry of the measurement location with respect to the rectangular loop, only three E/M calculations are required:

i. effect of current in DA at P;
ii. effect of current in BC at P;
iii. effect of current in one short side (AB, or CD) at P.

A simplification can be introduced to reduce the number of calculations from three to two, eliminating (iii) above. The rectangular loop has been replaced, in Figure 2, with a quadrilateral, ABDC, of the same cross-sectional area, where now the product a = d.(l.+L)/2 is set equal to rectangle's a' = d.l. The key point is that P now lies at the intersection of the projections of the canted short sides, BA, and CD. The magnetic field on the axis of a straight current element[1] is identically zero, and so the sides AB and CD play no further part in the calculation of the magnetic field at P. As P is moved along the x-axis, so the short sides AB and CD can be rotated through small angles to maintain P at the intersection of BAP, and DCP, etc. The lines BAP and DCP are also radials through P.

The benefit is that only two calculations are needed, for positive effect of the side DA, and for the negative effect of side BC. The only demerit is that the lengths are now unequal;

from similar triangles, $L = l.(x + d/2)/(x – d/2)$

If the target is still a strictly rectangular loop, then it can be modelled as a series of thin quadrilaterals, as depicted in Figure 3. Point G, if joined directly to A, re-constructs the top of the rectangle. To do that, the former angled side, AB, has been re-joined to point G, by means of one vertical, EF, and two radials from P, AE, and FG. If the process is continued, a rectangle results.

The vector magnetic potential, **A**, due to a small current element, **I.**d**S** varies as the reciprocal of (radial) distance from P, which means that current elements subtending the same angle from P exert equal contributions to **A** as measured at P. Hence the new section EF exactly substitutes for the former section GB, and so the irregular conductor AEFG is magnetically equivalent to ABG. Similarly, on the lower side, the serrated path from C1 to D exactly replaces the former corner at point C. That procedure can be reversed to turn a rectangular loop into a quadrilateral with two sides being radials through P.

The magnetic field intensity doesn't quite scale as **A** but it can be shown that for large x, the **H**-field at P is proportional to the current, **I**, and to the area enclosed by the loop. Ergo, any rectangular loop can be represented approximately by a quadrilateral loop with the same area. The condition that two sides be radials suggests the term *sectorial loop* for this particular geometry.

8.3. Analysis of H-field from elemental *sectorial loop*

8.3.1. Classical analysis

The d.c. and low-frequency a.c. magnetic field can be calculated for the sectorial loop by considering the effect of just the two "long" sides, DA, and BC. The vector magnetic potential, **A**, is linear and superposition may be applied. Let the loop in Figure 4 be considered to be in the x-y plane, with its centre at the origin (x = 0). P is on the x-axis, which also bisects the conductor DA, which is at x = d/2 and parallel to the y-axis. Hence, the value of **A** at P, **A** = $A(x_P)$ in the y-direction because **A** is always parallel to the associated current element, I.

The contribution to **A**, d**A** , due to the current element I d**S** is given [6] by:

[1] The isopleths (contours) of A are spherical shells (circles) and, on the axis, grad A points precisely along the axis of the current element. With the vector and its gradient parallel, curl A = 0.

$d\mathbf{A} = Kd\mathbf{S}/r$ (in y-direction) . . . .{ 1 }

where $K = \mu_0\, \mu_r\, I\, /4\pi$

The total value of A^+ due to DA, is twice times the integral from $y = 0$ to $y = l\ /2$ of such current elements,

thus $$A^+ = 2K \int_{\theta=0}^{\theta=\Theta} dS / r. \qquad \{2\}$$

where $\Theta = \arctan(l/2x)$

The rigorous derivation of the following Equation {15} can be found at Appendix B. For the sake of clarity, the numbering of the Equations is contiguous between main text and Appendix B.

$$H^+ = (dA/dx)/\mu_0\, \mu_r = K/\{\mu_0\, \mu_r\, x\sqrt{[4x^2/l^2 + 1]}\} \qquad \{15\}$$

The above result is for one half of the loop; the other half contributes negatively, reflecting the fact that the return current flows through BC in the opposite direction to that in DA. The difference (positive) is the total effect of the loop at point P.

The equivalent operations that led to Equation { 7 } (Appendix B) can be pictured as in Figure 5, where A^+ and A^- are the partial contributions to the total vector magnetic potential A_{LAT} (x) corresponding to the front (DA) and back (BC) current elements respectively. Close to each current element, the magnitude of | A | approximately follows a reciprocal distance law, as should be expected from inspection of Equation {1}. With DA and BC properly dimensioned, as explained in Section 9.2, the front and back contributions to **A** cancel exactly. Hence, somewhat surprisingly, $A_{LAT}(x) = 0$ when $x = x(P)$.

8.3.2. Part played by radial sides, AB and CD

The contribution to the potential $\{A_{RAD}(x)\}$ from the other two sides, AB and CD is shown near bottom of **Error! Reference source not found.**. Being radials through P, the vector magnetic potential $\mathbf{A}_{RAD}$ has a zero curl at P exactly, and | A | undergoes a minimum along the x-axis at x = x(P), i.e. $\partial A(x)/\partial x = 0$. (The absolute value of A_{RAD} is immaterial.)

Generally, $$\text{curl } \mathbf{A} = \{\textbf{grad } A\} \wedge \{\text{dir } [\mathbf{A}]\} \qquad \{16\}$$

$$= \mathbf{j}.|A| \sin \varphi \qquad \{17\}$$

where φ is the angle between dir [A] and the direction of grad A.

Along each of the radials through P, **grad** | A_{RAD} | is aligned in the same direction as $\mathbf{A}_{RAD}$ itself (dir [**A**] = dir [**I**] for the generating current elements), hence Equation {17} indicates that, since: $\sin \varphi = \varphi = 0$, then: curl A_{RAD} vanishes.

Wherever P is defined, for any sectorial loop, it must lie at the intersection of two radials, and hence in a null of curl $\mathbf{A}_{RAD}$. That analysis justified the argument (See Section 9.2) behind the construction of the sectorial loop.

An inspection of Figure 5 shows that the first derivative of A_{LAT} (x) does not lead to the expected behaviour of the H field at larger separations. That is simply because the radial components, which may be disregarded at P, become increasingly important at larger distances (x) where broadside projections can be taken of all four sides of the loop. At P, the first derivative of A_{LAT} (x) = does give the correct H(x) dependence [as long as x = x(P) is held constant].

In performing the curl operation at or around P, it must be kept in mind that

$$|\,\mathrm{curl}\ \mathbf{A}(x)\,| = \partial A\ (x)/\partial x \qquad \{18\}$$

and not $$|\,\mathrm{curl}\ \mathbf{A}(x)\,| = dA/dx \qquad \{19\}$$

Equation {19} allows the abscissa (x-value) of P to move as the intersection of radials through two sides of a loop that is also changing its shape. That that is invalid, is simply proven by the realisation that, by definition, A_{LAT} (x) = 0 at P (always, for any P). Hence, direct differentiation with respect to x leads to the trivial result that d[A = 0]/dx ≡ 0, with the counter-intuitive consequence that the H field is also zero everywhere along the x-axis. Clearly Equation {19} is invalid.

The actual situation is as depicted in Figure 6, where the portions of the graphs for A_{LAT} (x) have been plotted for three different choice of P (i.e. x_i = $x(P_i)$). Sketched below the graphs of magnetic potential, are truncated plan views of the three sectorial loops corresponding to P_1, P_2 and P_3. At each P_i the local value of A_{LAT} (x_i) = 0 but the slope of A_{LAT} (x_i) is patently non-zero. That slope is given by the formulation in Equation {18}, whilst, strictly speaking, Equation {19} merely repeats the x-axis.

8.3.3. Innovative ***partial-integration*** method

The result established in the previous Section, summarised in Equation {15}, is deceptively simple. It yields, a closed analytic expression for the hypothetical magnetic field intensity at any distance from one side of a sectorial loop. That result cannot be made consistent with any physical system in the universe because it ignores the fact of continuity of (electric) current flow; there must be a return path for the current flowing from D to A. Whilst contributing nothing to the magnetic effect, the radial sides take the current from C *via* D and A to B but it remains to complete the circuit by conductor BC. When "x" is replaced in Equation {15} by "x + d" for the back side, BC, of proper length l (x+d)/x, a negative contribution can be obtained and the difference between the two ought to yield the real field intensity. However, the result is very untidy, containing 4th order cross terms in x, l and d.

A further problem is that the delayed potential, necessary for calculating the near fields for high audio or low radio frequencies, cannot be derived from expressions for H, only from the vector magnetic potential, see Appendix B, Equation {7}. That expression contains the inverse hyperbolic function, arctanh [ ] or $\tanh^{-1}$[ ], and any prospect of obtaining the difference in a compact form can forgone from the outset.

An altogether more optimal approach is suggested by the realisation (from Equation {1}) that for a constant current parallel segments subtending the same angle from a point, P, contribute exactly the same amount of vector magnetic potential. In the sectorial loop, the opposing currents that effect total mutual cancellation of the intermediate components of the vector magnetic potential, A_{LAT} might do something similar for the **curl** operation.

In order to exploit that property, the integration over the finite conductor, DA, has to be suspended while partial cancellation of infinitesimally small components of the overall curl A from front and back respectively are allowed to interact.

Consider the top half of a sectorial loop OABM in Figures 4, 7 formed when the x-axis bisects the full loop DABC. For easing the algebra, let A have x-y co-ordinates (0,h) and B at (-d, H). The radial, r_{MAX}, from point P, at (X_P, 0) connects A and D, thereby requiring that:

$$H = h(X_P + d)/\ X_P$$

Corresponding to Equation {2} but now distinguishing between r_1 for the nearer conductor and r_2 for the rear conductor in the expressions for A_{LAT}:

$$A^{+} = K \int_{\theta=-\Theta}^{\theta=\Theta} dS / r_1 = 2K \int_{\theta=0}^{\theta=\Theta} dS / r_1 = \int_0^h dy / \sqrt{(X_P^2 + y^2)} \quad \ldots\ldots\ldots\ldots\ldots \{20\}$$

$$A^{-} = 2K \int_{\theta=0}^{\theta=\Theta} d\Sigma / r_2 = \int_0^H dy / \sqrt{((X_P + d)^2 + y^2)} \quad \ldots\ldots\ldots\ldots \{21\}$$

where "dy" has been used for the back conductor, BC as it has for the front conductor, AD, coincident with the y-axis.

Equations {20} and {21} are independent and, in principle, can be evaluated separately, from which (see Equation {3}) to give an expression for the total vector magnetic potential, $A_{TOT} = A^{*} + A^{-}$ but it cannot be differentiated in closed form.

The full (and protracted) derivation of the following Equation {32} can be read in Appendix C.

$$curl\, A_{TOT} = -2Kdh / X_P (X_P + d)\sqrt{X_P^2 + h^2}$$

Recalling Equation {8} and substituting $K = \mu_0 \mu_r / 4\pi$ (from Equation {1}) yields the general expression for the magnetic field:

$$\boxed{B_z = curl\, A_{TOT} = {-\mu_r \mu_0 I d h} \Big/ {2\pi X_P (X_P + d)\sqrt{X_P^2 + h^2}}}$$

The expression can be interpreted as showing the well-known behaviour of the static field, diminishing as inverse cube of distance ($X_P >> d$, or h when a long way from the loop), and roughly inverse distance when very close ($X_P << h$) to the nearer conductor (compare Ampère's Law). (Note: the expression approaches Ampère's Law precisely as the second and third terms in the denominator approach d and h respectively.)

For the general case, it is convenient to observe that the denominator comprises three terms:

- X_P the perpendicular distance from the nearer side (AD);
- (X_P + d) the perpendicular distance from the far side (BC);
- the hypotenuse distance from P to either the A or D extremity of the loop.

The factor 2 in Equation {31} records the fact that from Equation {20} forward, the integral has been performed over the "top" half the loop only; the total length of the near conductor being 2h. The cross-sectional area of the sectorial loop is, then, ½ (2h +2H)d = d(H+h). The sectorial loop approaches a rectangular shape when P is far removed. For P closer, it was shown in Section 8.2 how a sectorial loop may be replaced with a rectangular loop, for the same length of the near conductor (AD). Hence, the cross-sectional area becomes a = 2dh, and the term 2Idh represents the classical magnetic dipole moment, $m_z = Ia$ pointing in the z-direction, so enabling the formula (Equation {33}) to be written thus:

$$\boxed{B_z = {-\mu_r \mu_0 m_z} \Big/ {4\pi X_P (X_P + d)\sqrt{X_P^2 + h^2}}}$$

8.3.4. Frequency Range

In one dimension, the Liénard-Wiechert retarded potential at the origin, from a set of stimuli Q_i at x_i, at time t, is essentially of the form,

$$Z(0, t) = \Sigma_i Q_i\{x_i, \mathbf{[}t - x/c\mathbf{]}\}/x_i \qquad \{34\}$$

where c is the speed of light (in vacuum). The emboldened square **[ ]** braces imply retarded time. It the i[th] stimulus, $q_i(x_i,t)$ has an oscillatory time varying form, thus:

$q(x, t) = q_0 \cos 2\pi ft$, where f is the frequency in hertz, and Equation {34} can be written:

$$Z(0, t) = \Sigma_i q_i \{\cos(2\pi f \mathbf{[}t - x_i/c\mathbf{]})\} / x_i \qquad \{35\}$$

Noting that $\lambda = c/2\pi ft$, is the wavelength associated with the propagation of any radio waves in free-space, the retardation can be rendered an explicit as a relation of two spatial quantities:

$$Z(0, t) = \Sigma_i q_i \{\cos(2\pi ft - x_i/\lambda) \} / x_i \qquad \{36\}$$

Inspection of Equation {36} indicates that the difference between the delayed potential and the (un-delayed) original stimulating function, is precisely the effect of the angular perturbation, $\alpha = x_i/\lambda$. When α is small, the difference is approximately $2 \alpha \cos(2\pi ft)$, which goes to zero as $x_i/\lambda = \alpha$ goes to zero. As long as $x << \lambda$ the effect of the Liénard-Wiechert retarded potential may be ignored.

If the current stimulus is a base-band digital data link, then the waveform will give rise to a spectrum of frequencies, being harmonics of the clock frequency, extending from d.c. (0 Hz) up to the value of the bandwidth. If the dominant modulation is OFDM, as in ADSL, then the overall bandwidth is roughly equal to the one-way channel capacity in Bauds. For example, a 1 Mbits/s/ ADSL link comprising 256 sub-channels of 4kHz would need the highest to be centred on 1022 kHz, with a 3 dB roll-off at 1024 kHz. From an RFI point of view, the worst spectral frequency is 1022 kHz. Whatever the modulation method, the higher the channel capacity, the higher the frequencies that will characterise the top end of the spectrum.

The traffic waveform is carried primarily in the differential-mode, or *balanced* mode, of the conductors comprising the power line. It is reasonable to assume that the balanced-to-unbalanced ratio can only get worse at the higher frequencies, i.e. more common-mode current flows for the same differential mode current. That effect tends to bias the spectrum of the coupled magnetic field towards its high-frequency tail. A second effect is the dramatic increase in radiation resistance with frequency (See Section 9) following a fourth power law. Ergo, electro-magnetic (radio) interference from both the near-field coupling, and far-field radiation is expected to be dominated by the just the top end of the spectrum of the base-band signal.

House-scale dimensions require λ to be at least hundreds of metres, implying an upper frequency limit of about 1 MHz. The foregoing analysis then may be used for the PLT effects of audio, and typically ADSL base-band digital traffic. For higher base-bandwidths, and for RLAN carriers in the short-wave band (2 – 30 MHz) the formulation is not valid, and a deeper analysis, based on the retarded potential should be required

9. High-Frequency Analyses

9.1. Strategy

In Section 8 the near field was considered to have the magnitude of the static field from a direct current (d.c.) but modulated at a.c. frequencies below the limit given in Section 8.3.3. That limited the scope to about 1 MHz, or 2 Mbit/sec using NRZ binary base band.

Different formulations are required to cover the radio-interference mechanisms at successively higher frequencies, and no single formulation works throughout. The relevant mechanisms that come into play are discussed below, before applying the various formulations in their respective frequency regimes.

As a first attempt to simplify a potentially very complicated analysis of an undefined "typical" scenario, the following method will be applied. The corresponding loop current will be predicted that just exceeds the field-strength minimum for HF broadcasting protection, 40 dBμv/m. Those estimates can, in principle, be worked up into a regulatory template for the upper limit on PLT-generated common-mode current.

9.2. Coupling and radiation distinguished

For all higher frequency bands, the Liénard-Wiechert retarded potential destroys the near cancellation of the magnetic influences of the nearer and farther conductors, when they are carrying a constant common-mode current. It has already been pointed out (Section 8.3.3) how the characteristic frequency is the highest in the base-band spectrum, and often numerically the same as the quoted bandwidth. One of the reasons is the increased balanced-to-unbalanced conversion in the wiring. The other is the increased efficiency of both coupling mechanisms, magnetic coupling in the near field, and radio-wave propagation into the so-called *far-field*.

The two effects are not the same and must be distinguished.

Magnetic coupling operates between two loops, loop 1 inducing an e.m.f. in the other, loop 2. As the frequency of an a.c. current of constant amplitude in loop 1 increases so the e.m.f. induced in loop 2 rises. The magnitude of the magnetic field (flux density, B, or magneto-motive force, H) through the second loop remains constant but the e.m.f. depends, through Faraday's Law, on its rate of change, and hence its frequency. The strength of the coupling decays rapidly, approximately as the cube of the separation distance, hence the term *near field*.

The *far field* is what generates the propagating radio-wave, and its influence on a receiving coil or other antenna is properly treated as a radiation of energy as electrical power. In that case, the received power is a function of the strength of the H-field only, given certain relations between distance, wavelength, and antenna properties. There is no need to consider any mutual coupling between proximate loops; radio waves are launched autonomously into the universe, whether or not any energy is subsequently harvested from it. The key point to note is that the efficiency of launching such radio waves rises dramatically with frequency. An equivalent statement is that, for a constant amplitude of loop current, the intensity of the propagating H-field rises with frequency. As in all simple radio waves, the H-field varies as the inverse of distance from the (virtual) radiating source.

The *near field* by definition decays much more rapidly with distance, r, and any Taylor expansion contains powers of r^{-2}, r^{-3}, r^{-4}, ... etc. but never as r^{-1}. Hence, however intense is the near field, it is always exceeded by the far-field's r^{-1} dependence at greater distances. A traditional rule of thumb among antenna engineers, for general electromagnetic fields is that the far field predominates only after about 6λ from a radiating element, and so for the case of a magnetic loop antenna, the near-field component is expected to predominate indoors for all radio frequencies of interest to PLT. Normal antenna formulas are not a valid alternative to the full rigorous treatment of Jordan and Balmain, and/or Liénard-Wiechert retarded potentials.

The near-field effect can, in principle, be calculated from Equations {33} and {34}, the second illustrating how the Liénard-Wiechert retarded potential opens up a wavelength-dependent (frequency-dependent) difference between the nearly equal but opposite terms in Equation {22}. However important that is for a closely coupled receiving loop or coil, the contribution from the far field may not be overlooked, since it contributes to the overall coupling and may even predominate at higher frequencies.

9.3. Determination of frequency regimes

At frequencies where the free-space wavelengths, λ, is comparable with the dimensions of the loop, it is reasonable to assume that individual lengths of wiring can act as monopole or dipole antennas, contingent resonances raising their efficiency to a level of dominance over all the other coupling mechanisms. Hence, in this *resonant-spur* regime, it is appropriate to realise that real wiring schemes contain, in addition to ring mains, spurs to isolated power extensions, both permanent and temporary (e.g. the ubiquitous mains-extension lead). It is thus easy to see how monopole antennas can develop, the different lengths picking our different bands of frequencies at which to resonate. It is even conceivable that half-wave dipoles or V-antennas should occur when two or more spurs are wired to a common point. The formulations for all those standard types of antenna have been thoroughly and widely documented in the published literature.

Less well documented is the current distribution over a mesh of lossy transmission lines, with or without spurs. Open-circuited spurs can act as resonant stubs, at the appropriate wavelengths, i.e. certain frequencies of the r.f. current injected into the ring main. By attaching such spurs randomly, along a length of transmission line strong frequency-dependent reflections at the points of attachment may be seen by the main line. The reflections set up standing waves, which considerably modify the current distribution. Hence, the simple model of a uniform a.c. current distribution along the ring main becomes invalid. Even without spurs, it is quite likely that standing wave patterns will develop at particular frequencies (wavelengths).

At other frequencies, travelling waves may occur, this time, destroying the phase synchronicity, assumed in the simple model. The appropriate analogy, for travelling waves on a conductor that is long in comparison, is the Beverage antenna. Radiation from that type of travelling-wave antenna is highly directional. It is not thought to be of much application below the top of the v.h.f. band.

NOTE: there are three wavelengths to consider now:
- the free-space wavelength, λ;
- the wavelength of the intended differential-mode PLT signal, determined by the dielectric properties of the insulation of the supposed twisted pair;
- the wavelength of the Gobau waves associated with the common-mode current on the outside of the mains cabling, probably a little slower than in free space.

The distribution of *ad hoc* spurs (e.g. mains extenders) is probably sufficient to furnish radiating elements at radio frequencies into the low v.h.f. band (e.g. a metre long extension cable acting as a quarter-wave monopole at 75 MHz). For the lower frequencies in the HF band, standing-wave patterns on the loops of the larger ring mains offer more appropriate candidates for the dominant radiators. For instance, a ring supplying the ground floor of a small terraced house might easily have a perimeter equivalent to a free-space *rat-race* of 30 m, supporting the following standing wave patterns:

No. of half-waves	wavelength, λ	radio frequency, MHz	Wave band designation
1	30 m	10	short-wave (HF)
2	15 m	20	
3	10 m	30	
4	7.5	40	v.h.f.
5	6 m	50	
etc.		etc.	

Table 1 Standing waves on ring main.

Clearly, the short-wave band overlaps a second *standing-wave* regime. The formulation will be based on the Jordan and Balmain [7] near-field analysis of a non-uniform current distribution along a half-wave dipole.

That leaves the third regime to cover the lower half of the short-wave band, where standing waves do not develop within individual houses. Uniform circulating current distributions can be modelled with reasonable expectation of predicting the interference to short-wave receivers. Allowing for the Liénard-Wiechert retardation, it is not the same case as short-range magnetic inductive coupling but true near-fieled from a large loop, and so it may be termed *loop radiation* to distinguish it from (magnetic) inductive-loop coupling on the one hand, and from the near-fields of the various non-uniform current radiators (standing waves) on the other.

The base-line *low-frequency* regime is where Equation {33} may be used, which covers Very Low Frequencies (VLF), Low Frequencies (LF), and the lower half of the Medium Frequency (MF) band.

9.4. Example calculations

9.4.1. Low-frequency regime

A small horizontal ring main is postulated, supplying an adjacent room, as illustrated in **Figure 8**. The measurement point, P, is one metre from the near lines, half way along the near wall of length 8 m. The criterion is that H not exceed –131.5 dB A/m, the plane-wave counterpart to a (horizontal) E-field of +40 dBμv/m.

The worst-case mains cable is assumed, i.e. one not enclosed in metal conduit. The PLT device induces a uniform common-mode a.c. current in the ring main that is everywhere phase-synchronous (no travelling waves).

At P, the loop current may not exceed 3.93 μA. (11.9 dBμA).

Alternatively, the amplitude of the loop current may not exceed 17.86 μA. (25.04 dB μA) for the same effect at Q, 3 m from the wall, as per the JWG emission limit.

The frequency range is from d.c. up to about 1 MHz.

9.4.2. Loop-radiation regime

The frequency range is from about 1 MHz to 10 MHz. *The full analysis has not yet been completed.*

9.4.3. Standing-waves regime

Jordan and Balmain [7] present a formulation for a straight dipole form of radiating element, generalised for an arbitrary current distribution that varies sinusoidally along the limbs, as a standing wave with velocity equal to the free-space velocity. The parameter is "I_{MAX}", the amplitude of the current at the peak of a standing wave, leading to the current distribution:

$$I(z) = I_{MAX}\sin(\pi/\lambda z)$$

Where z is the distance from the (open-circuited) end of the element.

The half-wave dipole is a special case of it but their formula can be used for short dipoles or cases where the total length, 2L, of the dipole is an odd number of half wavelengths. The transverse magnetic field at a point P is governed by three terms, all of which vary as the inverse of the displacement of P in a radial direction that is perpendicular to the dipole (i.e. P need not lie on the axis of symmetry). (Only the phasor terms, which determine the phase of the wave at P, depend on the direct distances from P to the centre, and two ends of the dipole.)

One of the three terms is sensitive to the current distribution and vanishes when the 2L is exactly half a wavelength. In that case, the feed point is exactly at the extremum of the standing wave ($\sin(\pi/\lambda z) = 1$) and so the feed current is "I_{MAX}". Since that term normally subtracts from the two other terms, it is clear that indicates that, for a given "I_{MAX}", the magnetic field peaks for a half-wave dipole. (It is not precisely a "resonance" because (a) the feed current is related to "I_{MAX}" through wavelength, and (b) the input impedance is also frequency dependent.)

Two minor surprises are that:

(1) the Jordan and Balmain formulation contains no explicit dependence of field strength (E or H) on length of the radiating element;
(2) there are no other terms beside the three that vary as reciprocal of distance, i.e. components of the "far-field";
(3) on the axis (beam) of the dipole, the three terms reduce to one. leading to a pure inverse-distance law for both "near" and "far" component of the H-field.

Hence, as far as EMC is concerned, the worst cases to consider are standing waves that simulate half-wave dipoles, or else fit 3, 5, 7 etc. half waves into the long (8 metres) side of the ring main in Figure 8.
The half-wave case best fits this, next-lowest, frequency regime.

As in Section 9.4.1 criterion that, one metre away, H should not exceed –131.5 dB A/m results in an upper limit to the standing-wave current of 1.65 µA. (4.35 dBµA). For a separation of 3 metres, the current limit may be raised to 4.95 µA. (13.9 dBµA). The amplitudes of the currents in the corresponding travelling waves are just over half the peak currents, allowing for losses in the line, rendering those limits 0.825 µA. (neg1.65 dBµA), and 2.47 µA. (7.9 dBµA).

The frequency range is from about 10 MHz to 40 MHz.

9.4.4. Resonant-spur regime

A small horizontal ring main is postulated, as before but now with two spurs into the target room, as illustrated in Figure 9. The short distance of the "1 m spur" from P, 2 m, is intended to cover the alternative case of an overhead line to a ceiling light, as well as the depicted lateral mains extender. Each spur is treated as a monopole whose electrical length is the same as its physical length, 1 m, and 2 m respectively.

The criterion is that either:
the (vertical) H-field not exceed –131.5 dB A/m;
or the horizontal E-field not exceed +40 dBµv/m.

The PLT signal is assumed to contain a broad enough band of frequency components to excite the resonances of both (all such) monopoles. At resonance the monopole acts as a sink for all the common-mode current, which can continue to be treated as the parameter of interest. A rough idea of the far field from the monopole can be gained by considering the equivalent, effective radiated power from an ideal loss less-quarter-wave monopole, with radiation resistance of 37.5 Ω. (The same field-current relation obtains in the illuminated hemisphere of a monopole as in all directions from a dipole, so generality is guaranteed.)

For the 1m spur at 2m separation, the H-field criterion is satisfied for a feed current not exceeding 3.294 µA.
In Section 99.4.3 it was pointed out that there are no other terms beyond the pure inverse-distance law, hence scaling to the other spur, 4 m from P, the corresponding threshold current for the 2m is 6.59 µA. The dimension of the spur is not important to the field relation, affecting only the radio frequency of the corresponding interference. Hence the 2m spur resonates in the region of 37.5 MHz, and the 1m spur at 75 MHz. (The Jordan and Balmain formula gives 3.31 µA for the same case.)

The point of measurement, normally related to the ring main/house wall is less appropriate here. For example, a ceiling light spur can be 2 m from a radio receiver below it but only inches from one on the floor above. The figures cited for the model scenario assume that the radio is 1 metre from a wall but 2 m and 4 m from spurs.

The frequency range is from about 40 MHz to 80 MHz.

9.5. Discussion of model results

The limits obtained above for the typical-house model have been summarised in Table 2. The variation with frequency regime is not large and it might almost be acceptable to interpolate for the missing Regime, "Loop Radiation".

Regime	r.f. range, MHz	Separation distance		Qualifications about current, etc.
		1 metre	3 metres	
Low Frequency	< 1	3.93 μA	17.86 μA	uniform
Loop Radiation	1 – 10	[TBA]	[TBA]	
Standing Waves	10 – 40	1.65 μA	4.95 μA	peak of standing wave
Resonant Spurs	40 – 80	< 3.29 μA	> 6.6 μA	monopole feed current, spurs at other separations.

Table 2 Common-mode current limits for E = +40 dB μv/m

The only simple regimes are the first two, where uniform loop currents can be assumed. Once resonances develop the choice of current parameter becomes a subject for discussion. As stated earlier, the peak standing-wave current is twice the corresponding travelling wave current, if that may legitimately be related to the out-of-balance current derived from the balanced PLT drive. It is possible that the generation of spurious common-mode current should be influenced by the wild variations in r.f. impedance along the standing wave pattern. Applying Kirchoff to the Resonant Spurs regime, the current at the feed-point of the monopole is the algebraic sum of the currents in the ring main on either side of the spur. If they should flow in opposition, then the monopole current could be twice the current in the ring. Conversely, the PLT current limit should then be half the value in Table 2

10. Implications for PLT bandwidth

Although not intended as Layer-1 technique for PLT, the ADSL standard makes a convenient baseline for comparison. ADSL employs OFDM in 256 sub-channels of 4 kHz each. With suitable QPSK or more sophisticated modulation standards, that bandwidth can support a capacity of about 1 Mbit/s at a modest signal-to-noise ratio.

If the ADSL data pipe were transcoded to similar modulation with different parameters, e.g. 113 @ 9.06 kHz, then a simple one-to-one correspondence with Digital Radio Mondiale (DRM) may be facilitated. The physical-layer bounds on the PLT channel are first predicted for a 0 dB signal-to-noise ratio at the input to the DRM receiver, 1 metre from the power line (ring main, as in Section 9). From Table 2 a common-mode current of about 4 μA seems acceptable.

A ratio of balanced to unbalanced current is 20 dB is assumed, allowing a PLT sub-carrier amplitude of up to 40 μA. The characteristic impedance of comparable D1 field telephone wires is about 120 Ω, so making the associated transmission power to be 192 micro-watt, or –37.17 dBm

The noise background is assumed to be determined by the r.f. noise on the power line itself. One way to treat it is to attribute a generous noise figure to the PLT receiver, e.g. F_N = +30 dB. Thermal noise is –174 dBm per hertz, so in a sub-carrier bandwidth of 9 kHz, the thermal noise is:
39.57 – 174 dBm = -134.43 dBm

The budget can now be performed, relative to a single 9kHz channel:

Let there be R dB signal-to-noise ratio at the DRM receiver, that the PLT transmission power must be reduced by R dB relative to the limit calculated for 0 dB signal-to-noise ratio, leaving a D dB for the PLT transmission:

R + D = (-37.17) –345 (-134.43) = 97.26 dB

If the DRM radio service is allocated R = 40 dB, there is still another 57 dB for the PLT channel. Shannon's famous expression,
$C = W\ (1 + \log_2(SNR))$
indicates a capacity of 9000(1+18.9) bits/s = 180.22 kilo-bit/s

Adding up 256 similar OFDM sub-carriers gives an overall capacity of 46.137 Mbit/s.

Adding a further 10 dB to the noise figure of the power line, so that FN becomes +40 dB, then the impact on PLT would be a reduction to 38.3 Mbit/s, still very adequate. The above relationships are illustrated in Figure 10.

11. Discussion

11.1. Summary

This paper contains some possibly original work on the calculation of general electromagnetic fields from typical rectangular loops on the scale of domestic wiring. From that point on various assumptions have had to be made about the transmission characteristics of imperfect two- and three-conductor power lines, since the whole issue of PLT radio interference stems from contingent imbalances in a system that was never designed to carry high-bandwidth information.

This Sub-Section discusses how to treat those assumptions, and weigh them in the context of conflicting claims that, on the one hand, PLT installations might behave better, or on the other, that they might turn out to be worse in practice. The technical points at issue will be no doubt include some or all of the following:

- assessment of the limiting H-field equivalent of the ITU-R standard [5];
- the range of the characteristic impedances of various power lines installed in the housing stock;
- the ratio of unbalanced (common-mode) current to the normal differential-mode current in a PLT transmission through existing power-lines;
- the validity of approximating electrical perturbations prevailing on domestic power lines, particularly in the r.f. bandwidth of PLT transmissions, to Additive White Gaussian Noise AWGN);
- the ability of PLT modulation and coding to improve the theoretical relation between signal-to-noise ratio and communications capacity;
- the facility locating receivers, whether portable or fixed, at least one metre away from (often concealed) the nearest power line.

Some parameters require further investigation, possibly based on statistics. They are:
- actual equivalent noise figures for domestic (low-voltage mains) power lines;
- actual noise statistics (APD) in the MF and HF bands;
- characteristic impedances and loss factors for standard low-voltage mains cables;

Many interactions are anticipated between emerging PLT techniques, ways of achieving EMC between them and emerging digital broadcasting standards, and all mediated through legacy and futuristic house-wiring conventions. There is room to optimise PLT techniques through compromise, embedded flexibility to improve trade-off situations, and innovative measures to suppress harmful radio interference.

11.2. Engineering base-line assumptions

11.2.1. Assessment of the limiting H-field

The ITU-R [5] have recommended acceptable minima for the broadcast service in terms of the electric field strength incident at the receiver. Extraneous noise and interference may not degrade the reception quality at that threshold in the public-service bands. The ITU have also defined various levels of service quality ranging up to 50 dB in terms of signal-to-noise ratio. The adoption of the ITU-R reception criteria implies a corresponding maximum to the electrical disturbance generated power-line broad-band traffic, which is believed to be primarily manifested in the ambient H-field. There is a technical point of difference between the ITU-R criterion expressed in terms of an E-field, and the PLT's main product, an H-field.

In the broadcast radio waves, the E and H field components, regardless of plane of polarisation, are related in a constant way through the so-called characteristic impedance of free space, Z_0. = 377 Ohm. Hence it is reasonable to re-cast the ITU-R criterion in terms of a minimal-service, H-field threshold. (NB the Z0 concept is true only for plane waves, i.e. waves that have emanated from a distant source; when very close to a radiating element, e.g. a half-wave dipole, the constant relation between E and H no longer obtains.)

Small portable radios often contain a ferrite-rod antenna, and that type of antenna senses the magnetic (H) field directly. Hence the H-field version of ITU-R criterion is particularly pertinent in the "medium-wave" (MF) and short-wave (HF) bands. Whilst a short active whip antenna could be postulated, the established[1] practice for receivers of medium and short wave bands has been for loop or ferrite rod antennas that sense the H-field directly. All such antennas have maximal sensitivity in the plane of the loop and two nulls in the polar directions of the loop or rod axis.

A potential recourse for the dedicated radio buff, is to attempt to orient the loop or ferrite rod so that its equatorial plane is perpendicular to the nearby mains wiring, so as to be insensitive to the H-field from the PLT current. In the case of a typical ring main, the corresponding PLT current is horizontal and so the loop axis will be oriented parallel to the dominant wiring run. In the unlikely contingency that the radio broadcast happens to be incident in the direction parallel to that wiring then the attempt to avert PLT interference will also place a null on the wanted signal. Removal to another room will usually permit a new PLT-free orientation, so restoring good broadcast reception.

With due care and attention on the part of the user, the foregoing scenario could protect medium-wave reception, since MF broadcasters use vertical polarisation, the H-field therefore being horizontal. The situation cannot be extended into the HF band, where sky-wave propagation modes are used, for example in the broadcasting of DRM in the short-wave broadcast band. The plane of polarisation is seldom constant, the signal susceptible to fading, and restriction on the free orientation of the receiving antenna due to PLT interference will be most unwelcome.

11.2.2. Characteristic impedance

The mains wiring is treated as an imperfectly balanced transmission line for the purposes of the PLT signal, which, in its broad-band manifestation, is modelled as radio-frequency wave. Thus the characteristic impedance, Z_0, precisely relates the voltage and current in the PLT wave. It is reasonable to attribute similar current levels corresponding to the lower frequency components of PLT, even whilst they cannot propagate as travelling waves. Hence, the prevailing PLT current is related to the power in the PLT signal through the characteristic impedance.

From a brief consideration of communications theory, the capacity of any channel is related to the ratio of the energy per bit, E_B, to the noise power spectral density, N_0, or else to the total signal-to-noise ratio, see Section 10. The noise power is held fairly constant by a number of natural and man-made factors, and so communications capacity, often in restricted bandwidth, is governed by the power, R, in the PLT signal. In turn, the current $< i >$that flows in the mains wiring is determined by the quotient of the PLT power and the characteristic impedance, hence $< i > = R/Z_0$.

Over-estimating Z_0 can result in under-estimating the signal current for a given PLT system. The low-voltage Wiring Regulations [8] do not anticipate the use of domestic mains for transmitting radio frequency signals. Hence mains wiring is not manufactured to a specific characteristic impedance, and the manner of installation does not pay any respect to transmission-line theory. Conventions also differ; power circuits in the UK have required three conductors, "Live", "Neutral" and "Earth", whereas, prior to 1966, lighting circuits were required to provide only "Live" and "Neutral" conductors[2]. An estimate has to be ventured and a reasonable

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[2] Since 1966 the 14th Edition of the Wiring Regulations [8] have required the provision of the third, "Earth" conductor for lighting, i.e. all, circuits.

comparison may be made with D10, a loosely-twisted, field telephone cable which has Z_0 = 120 Ω. The larger conductor separation in typical 30A power cables indicates a higher value of Z_0 but the larger diameter of the conductors and the almost continuous encapsulation by a PVC dielectric certainly lower Z_0. (Note: the common practice of sandwiching the "Earth" conductor between the "Live" and "Neutral" in plastic power cable, does not affect Z_0 directly but there are other consequences, see Section 11.2.3).

With the mandated installation of three-conductor wiring post 1966, some PLT systems work on the "Earth" and "Neutral" conductors. In such cases, the above remarks still apply, *mutatis muatandis* but there are further r.f. consequences, see Section 11.2.3.

A side benefit of the PVC dielectric is its loss factor at higher frequencies, which helps to attenuate the transmission of PLT signals and damp any contingent r.f. resonances.

11.2.3. Current (common-mode) imbalance

Low-voltage, domestic house wiring is dominated by the ring-main technique, where a number of power outlets, for example, *A*, *B*, *C* and *D*, are run in parallel from a loop of power line, supplied from location *E*. If power is drawn from one point, e.g. *C*, two current paths to *C* are *E-A-B-C*, and also *E-D-C*. The actual ratios of power currents will depend on the power load and from where it is being drawn. If there are slight imbalances in the d.c. resistances between the "Live" and "Neutral" conductors running from *E* to *A*, and from *E* to *D*, then a current loop may develop. That effect can be exacerbated at radio frequencies where, rather than d.c. resistance, the r.f. impedance to infinity between the "Live" and "Neutral" conductors is unbalanced by several factors:

- Different separations between the "Live" and "Neutral" conductors from the "Earth" conductor (as altering their respective impedances to infinity).
- Some older lighting circuits used single-pole switches, acting only on the "Live" conductor. The un-switched length of "Neutral" can then act as an r.f. stub when the other conductor has been effectively shortened. At particular radio frequencies that stub will resonate, and at all frequencies its capacitance to infinity will unbalance the r.f. transmission line, promoting common-mode currents.
- Two-way lighting switches necessarily use single-pole switches at each end. When the light is off, there is either a long loop of "Live" of a long loop of "Neutral" conductor, severely unbalancing the receptive terminal of the ring-main.
- Modern switches for fixed (e.g. cookers) or ephemeral (appliance leads) power spurs are double-pole, and so shorten/extend the "Live" and "Neutral" conductors equally; for the asymmetrical PLT scheme, only the "Neutral" is switched, leaving the "Earth" always connected to the PLT signal, which sees a open-circuited r.f. stub, etc.
- Whereas in modern installations the "Live" and "Neutral" combination is reasonably symmetrical, at low frequencies at least, the alternative PLT scheme using the "Earth" and "Neutral" conductors is highly asymmetrical.
- The presence of the "Earth" conductor sandwiched in-between the "Live" and "Neutral" conductors of some flat-style power cables, tends to exercise a beneficial screening effect but that can be destroyed if there is as imbalance in the r.f. capacitances to "Earth" of the "Live" and "Neutral" inside in any appliances plugged-in (not necessarily even switched on)..
- Different "Live" and "Neutral" capacitance to ground on the primary side of isolation transformers in consumer appliances.
- Different "Live" and "Neutral" capacitance to ground coupled from secondary grounding of isolation transformers in consumer appliances.
- Wiring of legacy fuse boxes that results in unequal lengths of the "Live" and "Neutral" conductors.
- Possible unequal r.f. impedances of "Live" and "Neutral" conductors in inductive-type domestic electric meters.
- Special unbalancing effects at the branching points of resonant spurs;

As an example of the last, a spur from a ring-main may continently support a quarter-wave open-circuit resonance in the common-mode. The common-mode input impedance of the r.f. *stub* (at its branching from the ring-main) may be very low (close to r.f. short-circuit) in a very precise location. The individual "Live" and "Neutral" conductors are effectively capacitatively coupled to the common-mode *stub*, and its low input impedance. At the top of the HF band, the position of the null in the standing wave on the stub is very local, on the order of inches. Any lack of symmetry in the connexion of the "Live" and "Neutral" wires in the junction box effectively couples different r.f. impedances onto the ring-main.

A conservative 5% unbalancing effect has been assumed, whereby the common-mode current, the difference between the two imbalances in the "Live" and "Neutral", is 10% of the total PLT current.

11.2.4. Noise on domestic power lines

The parameter with the biggest uncertainty and probably largest variability in practice is the noise factor for the PLT receiver. The basic r.f. electronics can economically achieve a noise figure of 10 dB but the device noise most probably is insignificant in comparison to mains-borne noise from man-made sources. Modern appliance that can, if not adequately "suppressed", inject r.f. noise onto the house wiring include thyristor-controlled dimmer switches, speed controls for power tools, fluorescent discharge lamps (strip-lighting), and, of course, other PLT systems, whether installed in the same house or in neighbouring premises.

The plain fact is that no EMC standard for PLT can govern the contingent deployment by consumers of noisy appliances, or legacy products not covered by the EMC Directive. It might not be unreasonable to eliminate certain extreme scenarios as too hostile for PLT to be expected to work. An adaptive PLT system could be allowed graceful degradation of capacity under situations of exceptional mains noise. Looking on the bright side, periods of exceptional mains-borne noise, associated with the domestic use of power tools, are likely to be intermittent and short-lived.

11.2.5. Shannon communications capacity

The application, in Section 10, of Shannon's Theorem assumes that the noise in the PLT system is additive, Gaussian, and uniformly distributed in frequency. Additive White Gaussian Noise (AWGN) is a theoretical abstraction seldom encountered outside certain natural noise sources, and unlikely to characterise mains-borne electrical noise. However, various modulation and channel-coding schemes have been established for particular departures from AWGN, e.g. Reed-Solomon coding for impulsive noise. Ideally, adaptive coding schemes can switch to the most appropriate coding for the channel's broad noise statistics in real time.

The worked examples assumed the ADSL bandwidth, i.e. 246X4 kHz = 1024 kHz. By employing good roofing filters the issue of PLT radio interference to the short-wave band could be avoided entirely. However that raises the question of an EMC trade-off between broadcasting bands. The better protection could be offered to the MF(medium-wave) band by spreading the PLT energy over a wider bandwidth, i.e. into the short-wave band. Shannon clearly shows that signal-to-noise ratio may be exchanged for bandwidth whilst maintaining a given channel capacity.

Alternatively, signal-to-noise in the MF band may be left as it is while the bandwidth is increased, resulting in an overall enlargement of channel capacity. The increase, in this case, is in direct proportion to the bandwidth-scaling factor (but see Section 11.4). Modulation techniques such as OFDM even allow different signal-to-noise ratios in different parts of the spectrum, i.e. shaped power-spectral density (p.s.d.) mask.

11.2.6. User considerations

The choice of the target location, P, one metre from the straight side of the rectangular current loop (See Figure 8) is considered to reflect the practical requirement. The typical radio listener positions his/her radio on top of an item of furniture, usually against a house wall. In many cases, the radio will be with a fraction of a metre from the wall, and at a height of less than one metre from the ring-main at skirting-board level, which does not make an oblique distance of one metre over conservative. The very relaxed ETSI draft specification of 3 metres (location Q in Figure 8) is deemed absolutely impractical; many UK homes have small rooms.

Away from lighting spurs, the ring main of the next story/flat will be less than 3 metres away from likely radio locations in the room beneath. Again, ceiling lights run out along spurs that cannot be much more than 2 m above most practical surfaces. Away from the lighting spurs, the ring-main of the next story/flat above will be everywhere less than 3 m from likely radio locations in the room beneath.

As to re-orienting the ferrite-rod or loop antennas to minimise radio interference from PLT, it has already been suggested that the need to optimise the vagaries of short-wave reception must take precedence, and having to meet a double requirement will be seen to be a grave disbenefit.

11.3. Suppression techniques

Assuming that a certain target PLT channel capacity required in a particular radio bandwidth is assessed as likely to infringe the criterion advanced in this Paper, the following remedial measures could be considered, referring to a basic PLT system:

(1) reduce the PLT power to a level acceptable for RFI, and accept unavailability when the mains-borne noise is above a lower threshold (see 11.2.4);
(2) increase the system bandwidth and exploit the Shannon's trade-off between power and bandwidth (see 11.2.5);
(3) as (2) but using a partitioned spectral mask, giving preferential protection to certain sub bands;
(4) applying discrete band-stop filtering (notches) to shield precise radio channels, either factory-fixed, or manually programmable, or dynamically adaptable (as in Cognitive Radio, SDR);
(5) offsetting clock frequencies to avoid spectral lines of the PLT interference falling on the international AM broadcasting carrier raster;
(6) dithering clock frequencies to avoid obvious heterodynes when spectral lines in the PLT interference beat with wanted radio carriers, or, for example, OFDM sub-carriers in DRM;
(7) sophisticated means to estimate the real-time common-mode imbalance and reign-in the PLT current as appropriate (this could make use of the fact that there will always be more than one active PLT device connected and that a special protocol could allow for joint estimation of Common-mode leakage);
(8) sophisticated means to detect the local-oscillator emissions from in-house radio receivers, and, from a data base of common first *intermediate frequencies*, calculate the carrier frequency to which the radio is currently tuned (the mains wiring itself could be used as a sort of "detector-van" sensing aerial). An adaptive notch filter is thereby immediately and automatically placed on that channel;

11.4. Scaling Factors

The PLT channel capacity can be scaled in direct proportion to r.f. bandwidth, for a given total signal-to-noise ratio. Since, in any well-engineered system, the noise is likely to increase with bandwidth, the overall PLT power level also has to be raised. Alternative a slightly slower rate of increase of capacity with bandwidth has to be accepted.

Raising and lowering the PLT power, for constant bandwidth has a direct (logarithmic) effect on theoretical (Shannon) channel capacity for AWGN. For other, more impulsive types of mains-borne noise, the capacity gain is less efficient. Equally effective for capacity in a fixed bandwidth, is the lowering of the noise, since it is signal-to-noise ratio that is the governing factor.

A less obvious parameter is the wiring imbalance, converting harmless differential-mode PLT current into common-mode current that does the harm. The modelling assumed 10% conversion but a trade-off can be posited; keeping the level of PLT-generated radio interference constant, the PLT signal power may be elevated inversely with respect to the current imbalance. Thus, an amelioration from 10% imbalance to 5%, would enable the PLT power to increase by a factor four (6 dB).

11.5. Optimisation benefits everyone:

The final optimisation relates to an assumption which has been taken for granted until now, that is the use of optimal channel coding. The predictions of the Shannon equation, based as it is on the premise of a theoretically ideal coding, cannot be realised without the best coding (and modulation) matched to the channel. Put negatively, if an inferior coding scheme is employed, then the expected channel capacity shall not be attained, and inevitably, capacity will be sought be brute-force methods such as increasing signal power, or bandwidth, or both. Were that to be permitted, the consequence of that should be an undesirable increase in radio interference.

Hence, the optimisation of the efficiency of the PLT communication system itself leads to the best use of the resources of spectral bandwidth and signal power so lowering the upward pressure on both. That is good news for radio reception too, since it will permit and even encourage the general reduction PLT signal power, and in turn common-mode currents. Such a movement was also in line with ETSI and ITU-R published policies on future use of the radio spectrum.

11.6. Standardisation issues [5]

Several tentative recommendations can be envisaged:

- advise guidelines for PLT deployment only over certain acceptable minimal standard of house wiring; institute a certification process based on house ages, NHBC etc.;
- anticipating future PLT roll-out, modify the Wiring Regulations [8] to anticipate use of mains wiring for PLT, specifying high-quality balanced power cable, with common-mode rejection ferrite collars at every junction and spur;
- revise the current draft ETSI standard to align with the ITU-R protection of broadcast service minimum. (see Section 9.1).

12. Further Work

The full analysis of the Loop-Radiation regime seems to require a semi-numerical method. If such a method has been done, then it will probably be parametrical and a spreadsheet will need to be constructed; if not, then a new analysis will be indicated. That too will be hybrid between partially analytical formulas and some numerical approximations to the difficult integrals. A finite element method could be amenable to an Excel spreadsheet means of solution, which would also allow an unlimited number of parameters.

More research into real-world house wiring installations will allow a tightening of the broad range of assumed values for the model parameters and so increase confidence in the revised results.

13. Conclusion

It is believed that the current lax limits being advised in ETSI are poised to endanger the long and successful enjoyment of millions of short-wave radio listeners to broadcasting in the AM (HF) bands. The advised upper limit to the emitted (radiated) field strength from domestic PLT is in explicit conflict with an ITU-R principle concerning protection of that radio reception. If from no other consideration than international harmonisation, that issue needs to be address and resolved.

This Paper has set out to calculate and recommend an acceptable PLT common-mode current, which has been related to a typical system performance profile for a well-engineered PLT system. Some parameters of the theoretical model need refining to which further research could usefully be directed. It is strongly believed that the best aspirations of PLT communications engineers can be reconciled with those of the broadcasting community.

Now is an important time to press for an aspirational benchmark standard, since once inferior systems are allowed to be rolled out, it will not be possible to rectify an unsatisfactory EMC situation, and all parties will be losers.

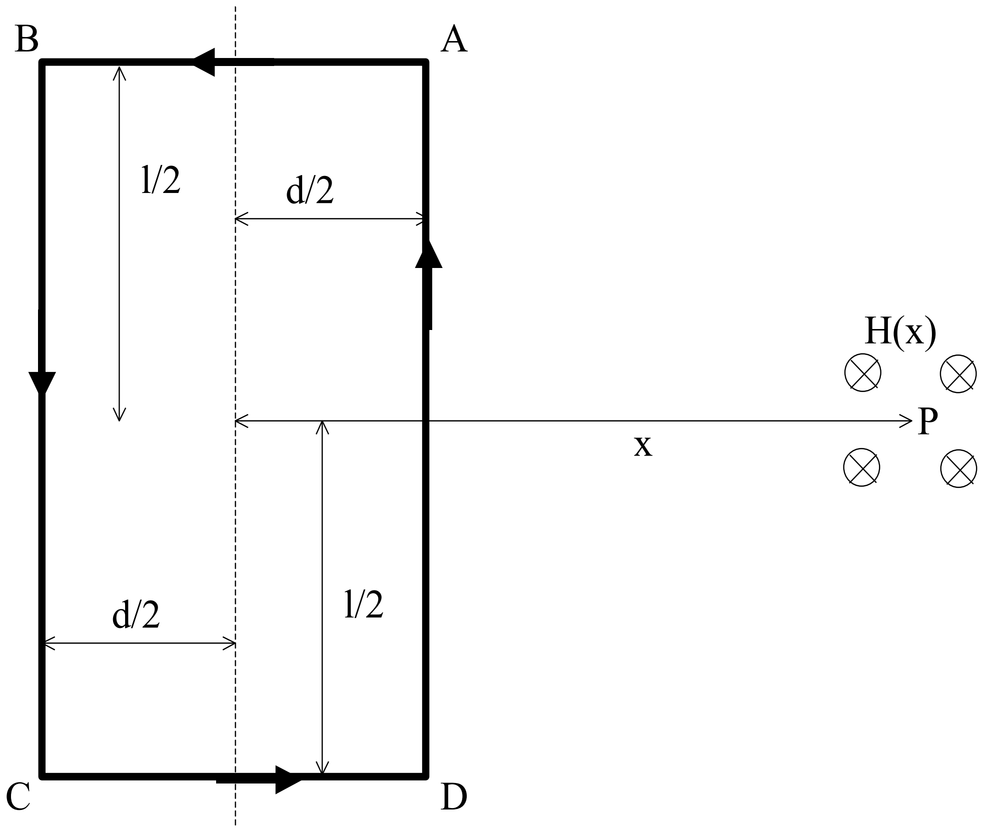

Figure 1 Elemental rectangular magnetic loop

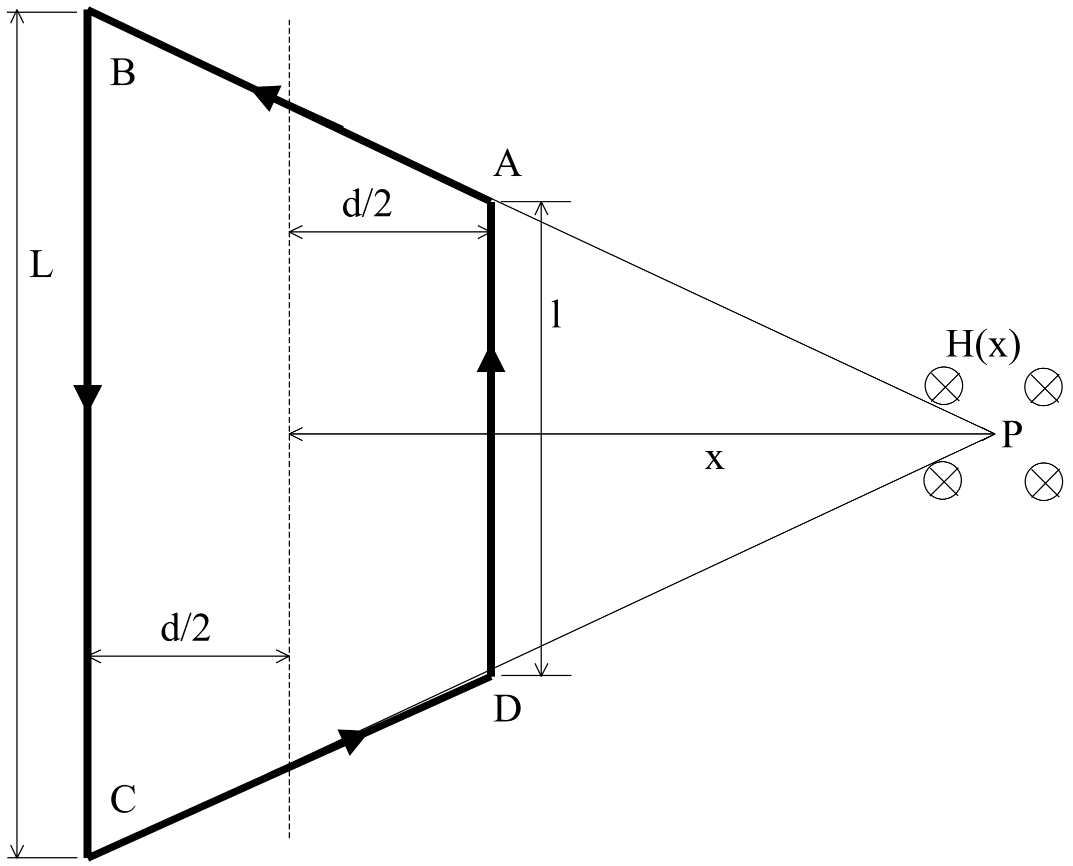

Figure 2 Simplified loop geometry for field calculation

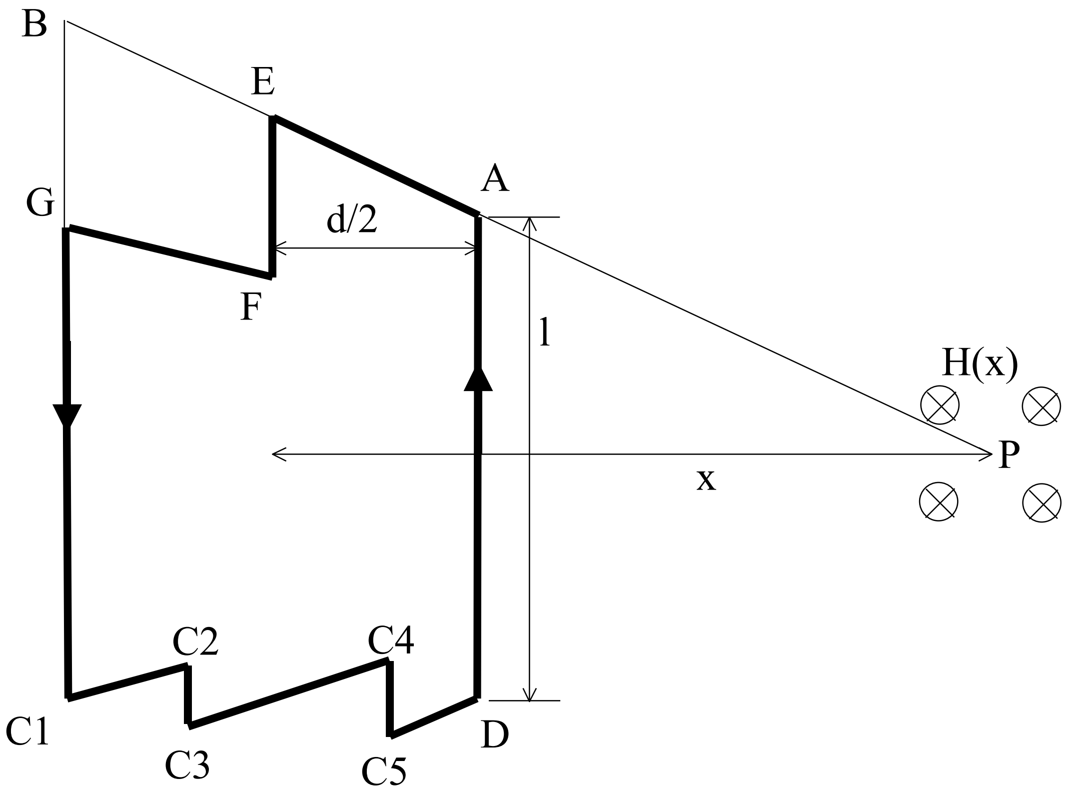

Figure 3 Synthesis of rectangular loop from elemental quadrilaterals

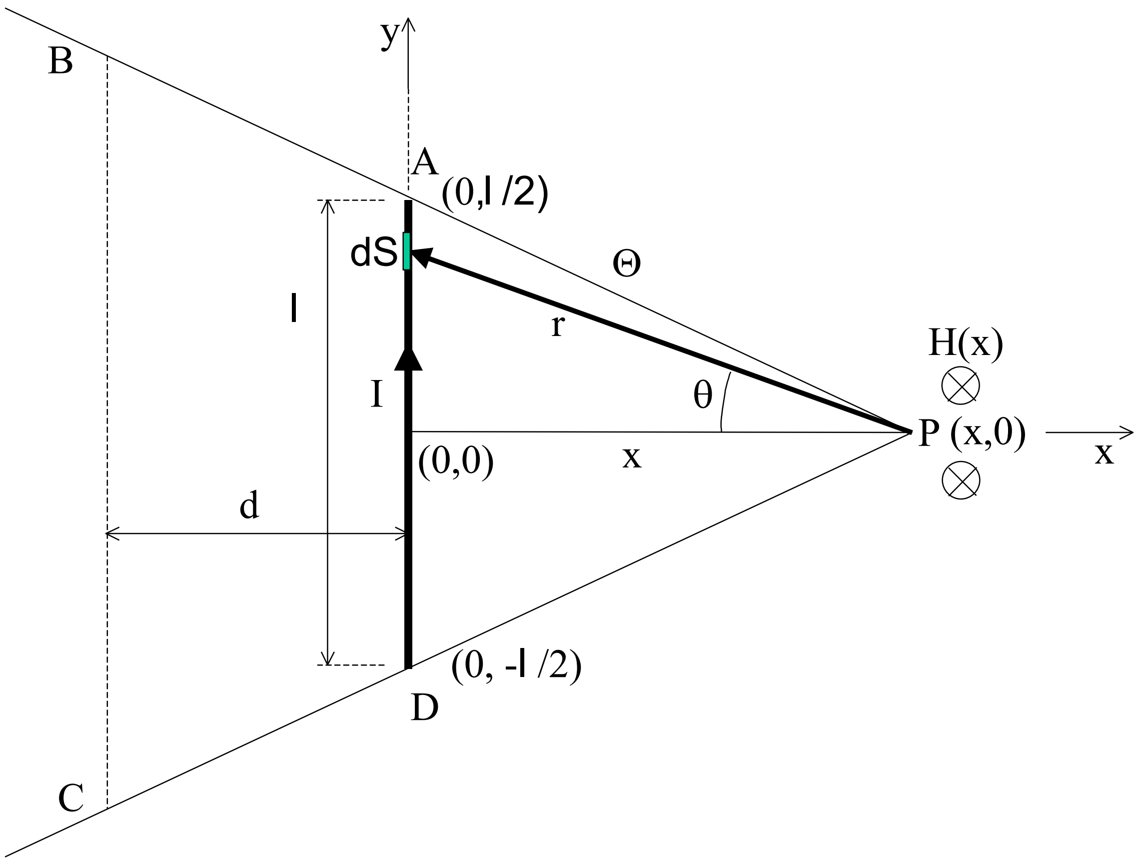

Figure 4 Calculation of partial vector magnetic potential at P

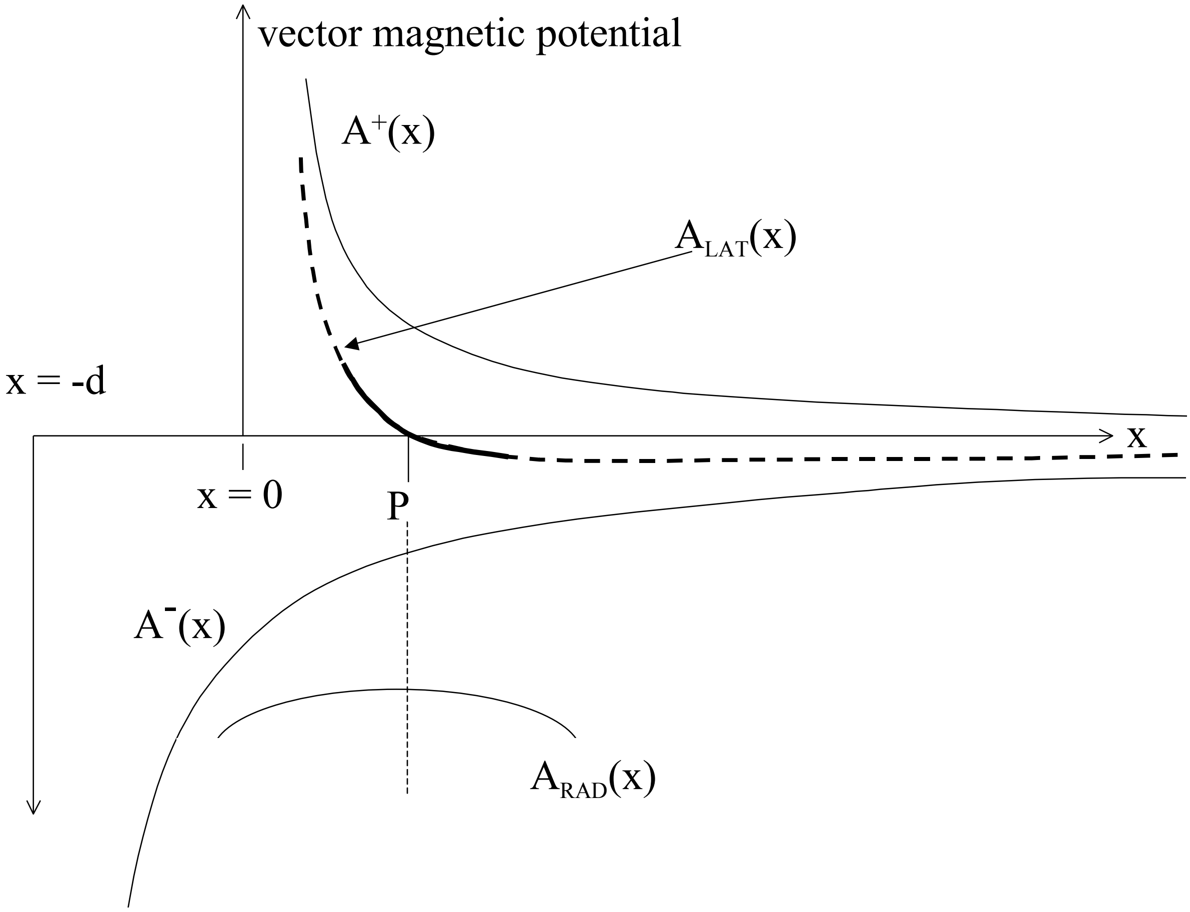

Figure 5 **Combining vector magnetic potentials from front and back elements of sectorial loop**

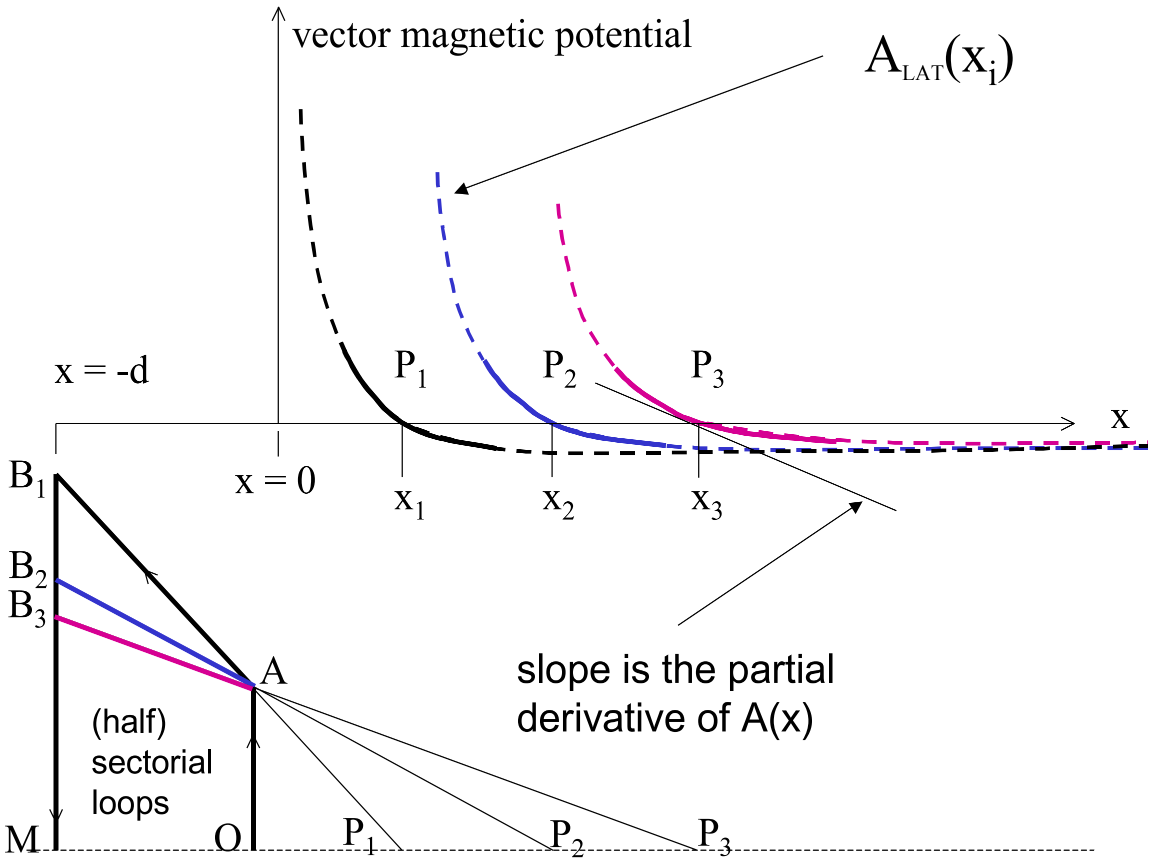

Figure 6 **Independence of (partial) derivative and absolute functional dependence of A(x)**

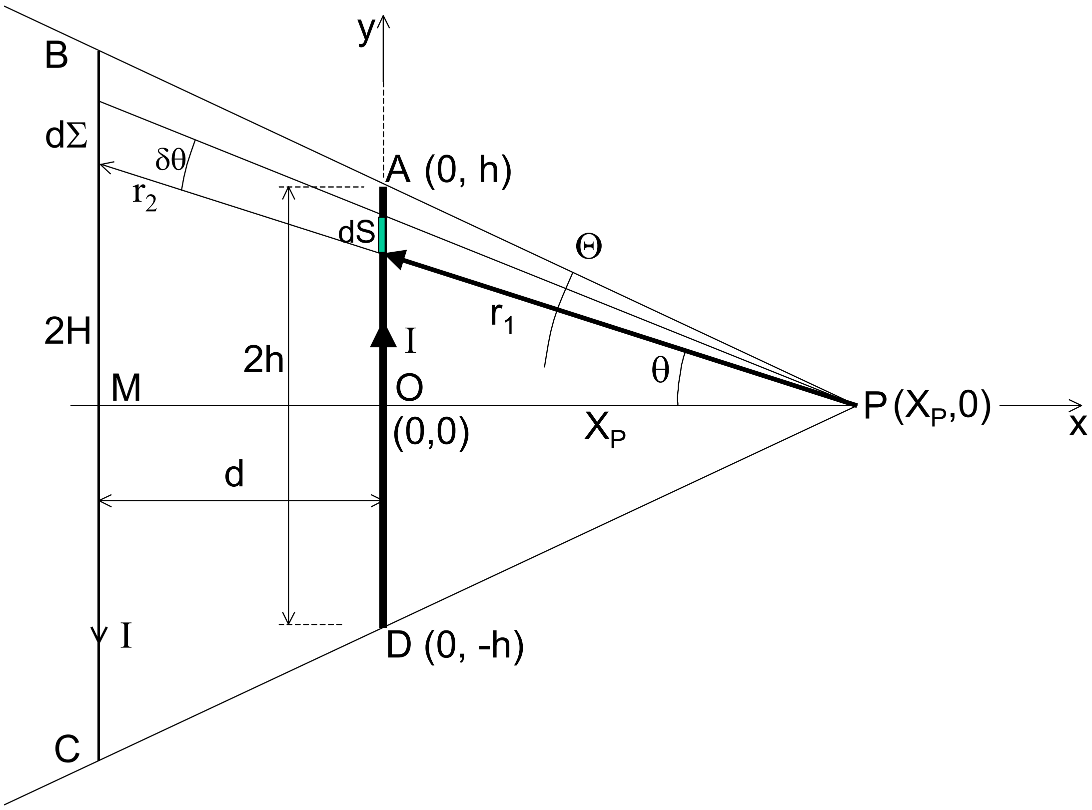

Figure 7 Geometry for method of partial integration.

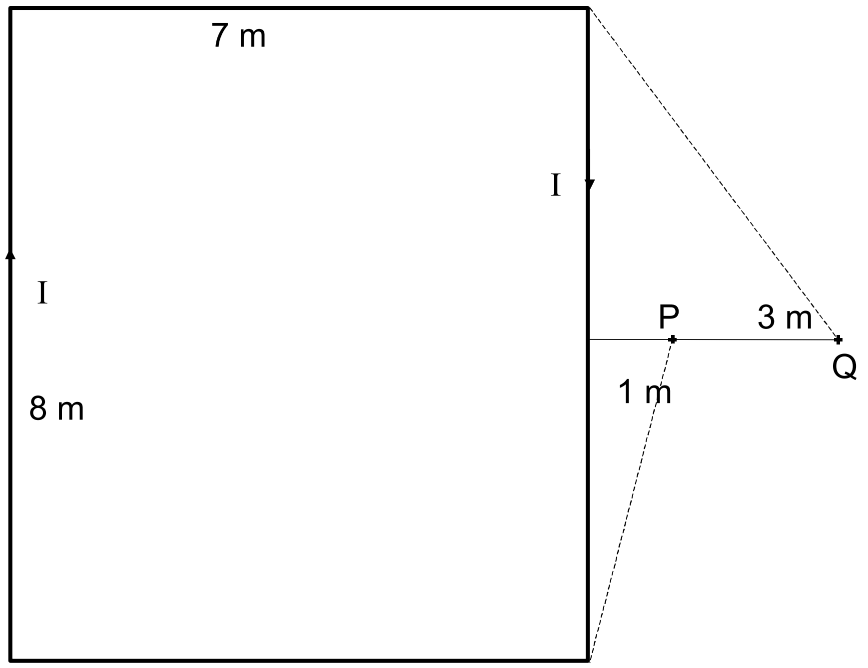

Figure 8 Room plan for low-frequency regime

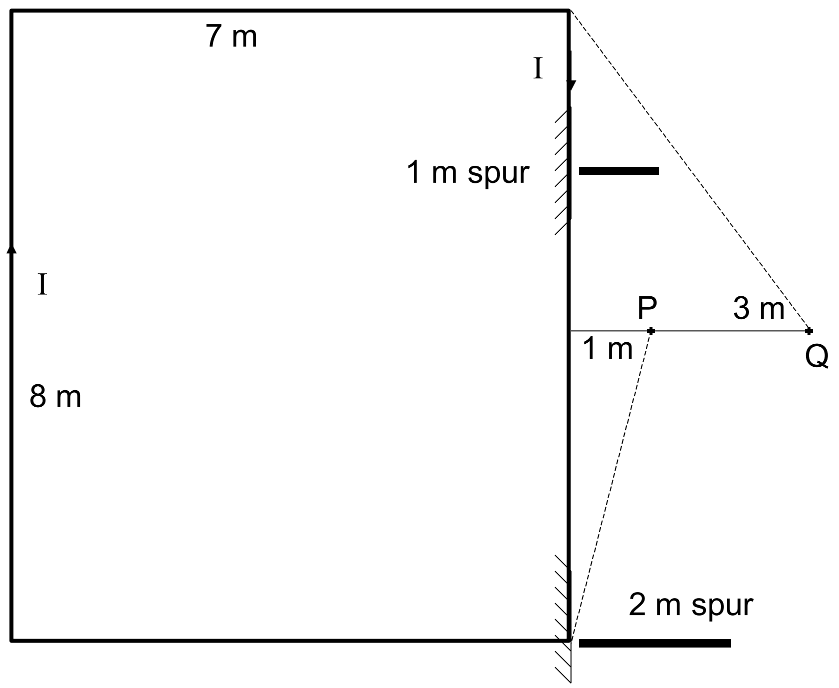

Figure 9 Room plan for resonant-spur regime.

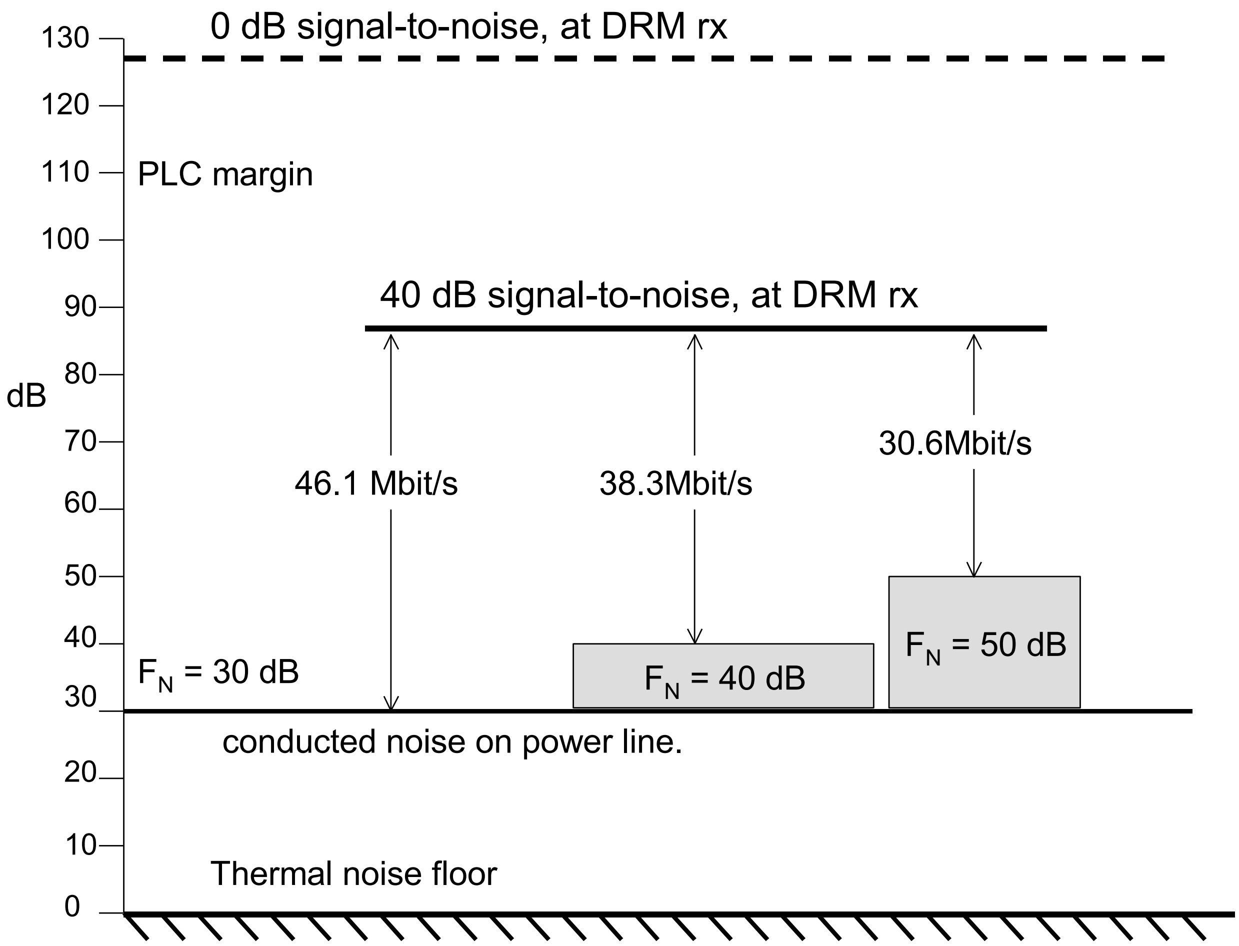

Figure 10 Trade-off chart: PLT capacity versus impact on DRM reception

APPENDIX A Technical Definitions and Abbreviations

For clarity, abbreviations and acronyms of standardising bodies and other authorities have been listed, in blue, ahead of the more general, technical abbreviations.

ARFA	Allied Radio Frequency Agency (supplanted by NATO FMSC)
a.c.	alternating current
ADSL	Asymmetric Digital Subscriber Line
ALE	Automated Link Establishment
a.m.	amplitude modulation/ed (noun/adjective)
AM	Amplitude Modulation (also closely associated with public
APD	amplitude probability distribution
ARQ	Automatic Request for Retransmission
AV	Audio-visual
AWGN	Additive White Gaussian Noise
BABT	British Approvals Board for Telecommunications (certification services)
BSI	British Standards Institute
BBC	British Broadcasting Corporation
BRAN	Broadband Radio Access Networks (ETSI)
b.e.r.	bit-error rate
CCIR	*Committe Consultatif Internationale* Radio (now part of the ITU)
CEN	European Committee for Standardisation
CENELEC	European Committee for Electro-Technical Standardisation (for standardisation of radio and TV receivers)
CEPT	European Conference of Postal and Telecommunications Administrations.
CISPR/a	(a can be A, B, I etc.) International Special Commission on Radio Interference (1934, overlaps IEC)
CDMA	Code -Division Multiple Access (a Direct Sequence method of outer coding)
CSMA	Carrier-Sensing Multiple Access (network protocol)
DTI	Department of Trade and Industry (UK)
DAB	Digital Audio Broadcasting (an ETSI-BBC Standard)
d.c.	direct current
DRM	Digital Radio Mondiale
d.s.b.	double sideband (modulation or spectral descriptor)
DPDT	Double-Pole Double Throw
DSL	Digital Subscriber Line
DS SS	Direct Sequence Spectral Spreading
EC	European Commission
ECC	Electronic Communications Committee (of CEPT)
ERC	European Radiocommunications Committee (CEPT)
ETSI	European Telecommunications Standards Institute
EU	European Union
e.i.r.p	effective isotropic radiate power
e.m.f.	electro-motive force
E/M	Electro-magnetic
EMC	Electromagnetic Compatibility
FDD	Frequency Division Duplex
f.m.	frequency modulation (generic)

FM	designation of public broadcast band (usually VHF) employing f.m.
HF	High Frequency (radio frequencies in range 1.5 to 30 MHz)
IEC	International Electro-technical Committee
I.E.T.	Formerly the Institution of Electrical Engineers IEE) the Instute of Engineering and Technology is the UK chartered body for all professional radio, electronics and power electrical engineers)
ISO	International Standards Organisation
ITU	International Telegraphic Union
ISDN	Integrated Services Digital Network
i.f.	intermediate frequency
ISDN	Integrated Services Digital Network
JEEC	Joint ECMA/ETSI Committee
JPEG	a video compression standard
LPRA	Low-Power Radio Association (UK)
LAN	Local Area Network
MAC	Medium Access Control (radio/telecommunications)
MCL	Minimal Coupling Loss
MF	Medium Frequency (strictly 300 kHz to 3000 kHz)
MFSK	m-ary FSK
o.e.m.	original equipment manufacturer
OFDM	Orthogonal Frequency-Division Multiplex
OFDMA	Orthogonal Frequency-Division Multiple Access
NHBC	National House Builders Certificate
NRZ	non-return to zero
PTT	Posts, Telegraphs and Telecommunications (administration)
PLC	Power-Line Communications (used interchangeably with PLT)
PLT	Power-Line Transmission (used interchangeably with PLC)
p.s.d.	power spectral density.
PSTN	Public Switched Telephone Network
PVC	Polyvinyl Chloride (a thermoplastic)
QAM	Quaternary Amplitude Modulation (a generic type of digital modulation standard)
QoS	Quality of Service
RSPC	Radio and Spectrum Policy committee of EC
r.f.	radio frequency
RLAN	Radio LAN (c.f. W-LAN)
RTT	Radio Transmission Technology
RTTE	Radio & Telecommunications Terminal Equipment (European Directive, effective 2005)
SDO	Standards Development Organisation
SDR	Software-Defined Radio
SPST	Single-Pole Single Throw
SRD	Short-Range Device
s.s.b.	single side-band
TDD	Time-Division Duplex
TDMA	Time Division Multiple Access
TV	Television (from τελοσ Greek. and *video* Latin) Russian-coined term

UN	United Nations
u.h.f.	ultra-high frequency (300 MHz to 3,000 MHz)
UHF	(UK military radio band somewhat misaligned with the SI convention)
UWB	Ultra-Wide Band (wireless data communication generic technique)
v.h.f.	very-high frequency (30 MHz to 300 MHz)
VHF	Very High Frequency; name of the public broadcast band (87.5 - 108 MHz in UK)
VLF	Very Low Frequency (strictly 3 kHz to 30 kHz)
VSWR	Voltage Standing-Wave Ratio
WRC	World Radiocommunications Conference
W-LAN	Wireless LAN
WT	Wireless Telegraphy

APPENDIX B **Full derivation of Equation {15}**

The derivation of the result quoted in Section 8.3.1 follows, with reference to Figure 4 in the main text.

The total value of A^+ due to DA, is twice times the integral from y = 0 to y = $l/2$ of such current elements,

thus $$A^+ = 2K \int_{\theta=0}^{\theta=\Theta} dS / r. \qquad \{2\}$$

where $\Theta = \arctan(l/2x)$ The solution in closed form is:

$$\begin{aligned} A^+ &= K[\arctan h(\tan(\Theta/2))] \\ &= K[\arctan h\langle \tan(\arctan[l/2x])\rangle] \end{aligned} \qquad \{3, 4\}$$

Let $\tan(\Theta/2) = t$

Let $g = \tan\Theta = 2t/(1-t^2) = (l/2x)$ . . . . {5}

Therefore $gt^2 + 2t - g = 0$

Solving the quadratic gives two roots, $t = -1/g \pm 1/g\sqrt{[1 + g^2]}$ . . . . {6}

Since t must be positive at all times, take the plus sign:

$t = -2x/l + \sqrt{[4x^2/l^2 + 1]}$

$A(x) = 2K\mathrm{arctanh}[2x/l\,(\sqrt{[1 + l^2/4x^2]} - 1)]$ . . . .{7}

The *partial* magnetic field at P due to the nearer conductor is given [5 – delete of ch to 4] by:

$B^+ =$ curl $\mathbf{A}$ (x) Webers/m^2 . . . . {8}

or in terms of magneto-motive force, **H**,

$H^+ =$ curl $\mathbf{A}$ $(x)/\mu_0\,\mu_r$, which because A(x) lies in the x-y plane, and varies only with x (in that plane), becomes:

$\mu_0\,\mu_r\,H^+ = \partial A(x)/\partial x = dA/dx$ . . . . {9}

Now if $Z = \tanh Y$, $Y = \mathrm{arctanh}[Z]$ which is of the form of the expression for A^+

and $dZ/dY = 1 - (\tanh Y)^2$ . . . . {10}

Hence $dY/dx = (dY/dZ).(dZ/dx)$

Substitute $Z = 2x/l \ (\sqrt{[1 + l^2/4x^2]} - 1) = -2x/l + \sqrt{[4x^2/l^2 + 1]}$ . . . . { 11}

whence $dZ/dx = 4x/(l^2\sqrt{[4x^2/l^2 + 1]}) - 2/l$ . . . . { 12 }

Since $Y = Y(Z)$ and is a function of Z alone, $dY/dZ = [\, dZ/dY\,]^{-1}$

Then $dY/dx = \{4x/(l^2\sqrt{[4x^2/l^2 + 1]}) - 2/l\}/\{1 - (\tanh Y)^2\}$ . . . . { 13 }

$= 1/\{2x\sqrt{[4x^2/l^2 + 1]}\}$ . . . . { 14 }

Substituting the physical quantities and constants,

$H^+ = (dA/dx)/\mu_0\,\mu_r = K/\{\mu_0\,\mu_r\, x\sqrt{[4x^2/l^2 + 1]}\}$ . . . . { 15 }

APPENDIX C Details of partial integration method (Section 8.3.3)

Consider the top half of a sectorial loop OABM in Figures 4, 7 formed when the x-axis bisects the full loop DABC. For easing the algebra, let A have x-y co-ordinates (0,h) and B at (-d, H). The radial, r_{MAX}, from point P, at (X_P, 0) connects A and D, thereby requiring that:

$H = h(X_P + d)/ X_P$

Corresponding to Equation {2} but now distinguishing between r_1 for the nearer conductor and r_2 for the rear conductor in the expressions for A_{LAT}:

$$A^{+} = K \int_{\theta=-\Theta}^{\theta=\Theta} dS / r_1 = 2K \int_{\theta=0}^{\theta=\Theta} dS / r_1 = \int_{0}^{h} dy / \sqrt{(X_P^2 + y^2)} \quad \text{.................} \{20\}$$

$$A^{-} = 2K \int_{\theta=0}^{\theta=\Theta} d\Sigma / r_2 = \int_{0}^{H} dy / \sqrt{((X_P + d)^2 + y^2)} \quad \text{............. } \{21\}$$

where "dy" has been used for the back conductor, BC as it has for the front conductor, AD, coincident with the y-axis.

Equations {20} and {21} are independent and, in principle, can be evaluated separately, from which (see Equation {3}) to give an expression for the total vector magnetic potential, $A_{TOT} = A^{*} + A^{-}$ but it cannot be differentiated in closed form.

A more fruitful approach is to synchronise the integration of Equations {20} and {21}, by realising that for every dS along OA, there is a corresponding $d\Sigma$ on MB, the common parameter is the angle θ and its infinitesimal angular shift, $\delta\theta$, such that $dS = r_1 . \delta\theta$ and $d\Sigma = r_2 . \delta\theta$.

Converting to a hybrid r-θ co-ordinate system, the two integrals can then be given common limits, 0 to Θ, and the two integrands manipulated inside the integral, thus:

$$A_{TOT} = 2K \int_{\theta=0}^{\theta=\Theta} [dS / r_1 - d\Sigma / r_2] \quad \text{.................................} \{22\}$$

From the geometry of similar triangles, $r_2/r_1 = (X_P + d)/X_P = dS(1 + d/ X_P)$, and similarly, $d\Sigma = dS(1 + d/ X_P)$ enabling a reversion to purely x-y co-ordinates, thus:

$$A_{TOT} = 2K \int_{\theta=0}^{\theta=\Theta} [dy / \sqrt{(X_P^2 + y^2)} - dy(1 + d / X_P)/(1 + d / X_P)\sqrt{((X_P + d)^2 + y^2)}]$$

$$A_{TOT} = 2K \int_{\theta=0}^{\theta=\Theta} dy[1/\sqrt{(X_P^2 + y^2)} - 1/\sqrt{((X_P + d)^2 + y^2}] = 0 \text{ . . . . .\{23\}}$$

That is the result that was obtained from inspection of Equation {7} in Appendix B. It corroborates the situation depicted in Figure 6, where A_{TOT} is always zero at any P that is located at the intersection of the radials. The ease of the above method of integration can be extended to the full solution of Equation {18} for front and back conductors, thus:

$$curl\, A_{TOT} = \partial/\partial x(A_{TOT}) = 2K \int_{\theta=0}^{\theta=\Theta} \partial/\partial x[dS/r_1 - d\Sigma/r_2] \text{ . . . . \{24\}}$$

The formulation of Equation {24} has been termed "partial integration" because the partial derivative of the original expressions for A^+ and A^- in the integrand can be integrated in a closed form. Between Equation {22] and Equation {23} a number of terms enter with the abscissa (x-) value of P explicit, namely X_P. While forming the curl of A(x), at P, it must be held in mind that the sectorial loop associated with a particular $P(X_P)$ is immobile, and so the value of $X_P = Xi$ that determine the shape of dimensions of the loop cannot be take part in the differentiation operation, hence the use of the partial operator, ?/?x.

For clarifying the analysis, let the independent variable, x, be the variable for the curl (= ∂/∂x) operation. Once a particular P = Pi has been determined, set $x = X_P = Xi$ but only x is variable thereafter. For additional clarity, write $G = r_2/r_1 = 1 + d/X_P$ treating it as a constant under the curl operation. The C integral may be considered, as before, synchronised with respect to angles subtended at P, such that $d\Sigma = GdS$.

The variable parts in Equation {22} then become Ct, as follows:

$$curl\, A_{TOT} = 2K \int_0^h \partial/\partial x[dy/\sqrt{(x^2 + y^2)} - dyG/\sqrt{((x + d)^2 + y^2)}]\Big|_{x=X_P} \text{ . . . .\{25\}}$$

$$= 2K \int_0^h dy\{ -x/(x^2 + y^2)^{3/2} + G(x + d)/((x + d)^2 + y^2)^{3/2}\Big|_{x=X_P} \} \text{. . . . .\{26\}}$$

Integration is with respect to y only, and so the qualification, $x = X_P$ may be substituted before evaluating the *dy* integral.

$$curl\, A_{TOT} = 2K \int_0^h dy\{ -X_P/(X_P^2 + y^2)^{3/2} + G(X_P + d)/((X_P + d)^2 + y^2)^{3/2} \}$$

Since $r_2 = Gr_1$ then $(X_P + d)^2 + y^2 = G^2 (X_P^2 + y^2)$, the second integrand can be simplified:

$$curl\, A_{TOT} = 2K \int_0^h dy\{ -X_P/(X_P^2 + y^2)^{3/2} + G(X_P + d)/G^3(X_P^2 + y^2)^{3/2}| \}$$

$$curl\, A_{TOT} = 2K \int_0^h dy\{ -X_P/(X_P^2 + y^2)^{3/2} + G(X_P + d)/G^3(X_P^2 + y^2)^{3/2}| \} \text{. . . . .\{27\}}$$

The integral may be evaluated in the r-θ system by substituting $r_1 = \sqrt{(X_P^2 + d^2}$
and noting that G = (X_P + d)/X_P

$$curl\,A_{TOT} = -2Kd/G\int_0^h dy/r_1^3 \qquad \text{.................. \{28\}}$$

By substituting: $r_1 = X_P\sec\theta$ the variable of integration can be changed to dθ, noting that $y = X_P\tan\theta$ and, $dy/d\theta = X_P\sec^2\theta$. Hence, replacing dy with : $X_P\sec^2\theta\, d\theta$

$$curl\,A_{TOT} = -2Kd/G\int_0^h d\theta.X_P\sec^2\theta/(X_P\sec\theta)^3 \qquad \text{................. \{29\}}$$

The limits are next replaced with φ = arctan (h/X_P) (replacing Θ with φ) for clarity).

$$curl\,A_{TOT} = -2Kd/GX_P^2\int_0^{\theta=\varphi}\cos\theta.d\theta \qquad \text{.................... \{30\}}$$

$$= -2Kd/GX_P^2\left[\sin\theta\right]\Big|_0^{\varphi=\arctan(h/X_P)} \qquad \text{................. \{31\}}$$

Clearly, if tanφ = h/X_P then $\sin\varphi = h/\sqrt{(X_P^2 + h^2}$

Ergo,

$$curl\,A_{TOT} = -2Kdh/X_P(X_P + d)\sqrt{X_P^2 + h^2} \qquad \text{................................ \{32\}}$$

In Section 8.3.3. Equation {32} is turned into the magnetic (H) field that we required to calculate.